# Tutorial on Ergodic Control @ ICRA 2024

May 13th, 2024
Yokohama, Japan
https://ergodiccontrol.github.io/

# WELCOME TO THE ERGODIC CONTROL TUTORIAL

Todd Murphey
Mechanical Engineering
Physical Therapy and Human Movement Sciences
Center for Robotics and Biosystems
Northwestern University

Northwestern | CENTER FOR ROBOTICS AND BIOSYSTEMS

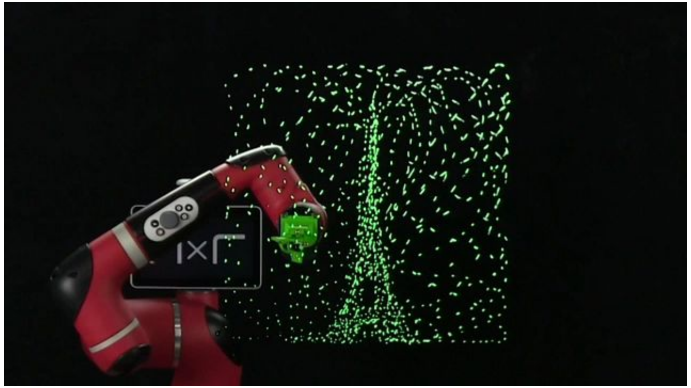
# Ergodic Control is multiple things

1. A measure from thermodynamics / statistical mechanics / information theory, connecting representations of robot trajectories to distributions
2. Specification of behavior, making some behaviors easier to specify
    1. An alternative to the use of trajectory error in the state space as
    2. Clarifies what the word ‘random’ means
3. A justification for using spectral representations, which have excellent scaling properties (e.g., scaling with respect to number of entities in an environment)
4. A design paradigm for robot behavior

# What We Will Learn Today

1. A tutorial on both ergodic metrics and ergodic control
    1. Assumes no background in ergodic…anything
2. How to use some standard implementations of ergodic control
    1. Starting with SMC, trajectory optimization, sampling-based methods, diffusion equations
3. Applications, applications, applications
    1. HRI, distributed search, manipulation, machine perception, reinforcement learning … robotics has a lot to gain from ergodic control
4. We will end with design approaches and current challenges.

# We Will Also Learn Other Things!



Murphey, 2018

DrozBot (Calinon, 2022)

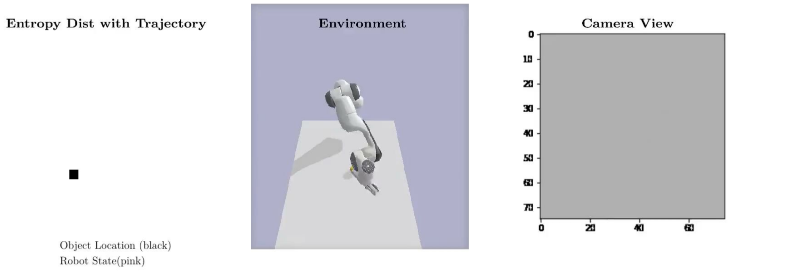
| | | |
|---|---|---|
| 08:45-09:00 | Welcome and Introduction | (Todd) |
| 09:00-09:15 | Why do we care about ergodic control? | (3-min pitches by organizers) |
| 09:15-09:35 | Technical introduction to ergodic control | (Katie) |
| 09:35-09:55 | Implicit Q&A | (Ian) |
| 09:55-10:00 | Teaser | (Guillaume) |
| 10:00-10:30 | Coffee break with posters presentations | |
| 10:30-11:00 | Sandbox codes | (Todd, Muchen) |
| 11:0-11:20 | Traj-Opt/Direct transcription | (Ian, Henry) |
| 11:20-11:40 | Ergodic control using diffusion, HEDAC | (Stefan) |
| 11:40-11:55 | Ergodic control in 1D, 2D, 3D and more! | (Sylvain) |
| 11:55-12:00 | Teaser | (Guillaume) |
| 12:00-13:00 | Lunch break (lunch boxes available) | |
| 13:00-13:20 | Sampling-based approaches | (Todd, Guillaume, Muchen) |
| 13:20-13:45 | Applications: HRI and biology | (Katie) |
| 13:45-14:10 | Applications: Search with UAVs | (Stefan) |
| 14:10-14:35 | Applications: Duration, reachability and optimality in exploration | (Ian, Henry) |
| 14:35-15:00 | Applications: Whole body exploration, insertion tasks, drawing, HEDAC on point clouds | (Sylvain) |
| 15:00-15:25 | Applications: Runtime robot learning of neural networks for perception and RL | (Todd) |
| 15:25-15:30 | Teaser | (Guillaume) |
| 15:30-16:00 | Coffee break with posters presentations | |
| 16:00-16:20 | Multi-Robot Coverage | (Guillaume) |
| 16:20-16:35 | Learning ergodic heterogeneous distributions | (Ananya, Guillaume, Ian) |
| 16:35-16:50 | Design considerations | (Todd) |
| 16:50-17:00 | Closing Remarks, Grand Challenges | (Todd, Sylvain) |

# MURPHEY ERGODIC PITCH

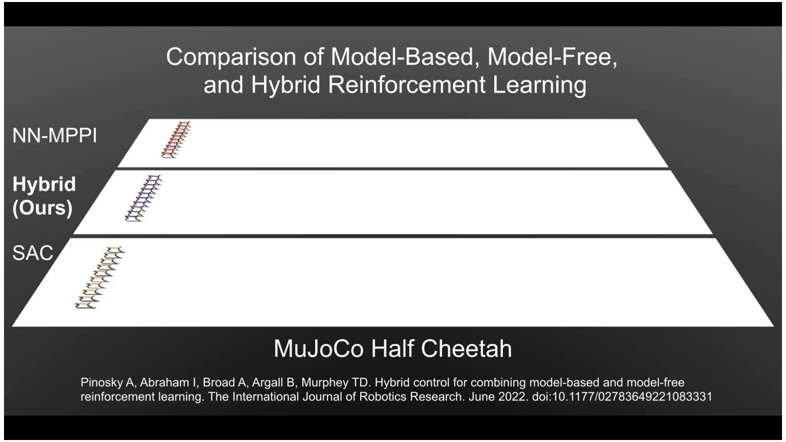
Todd Murphey
Mechanical Engineering
Physical Therapy and Human Movement Sciences
Center for Robotics and Biosystems
Northwestern University

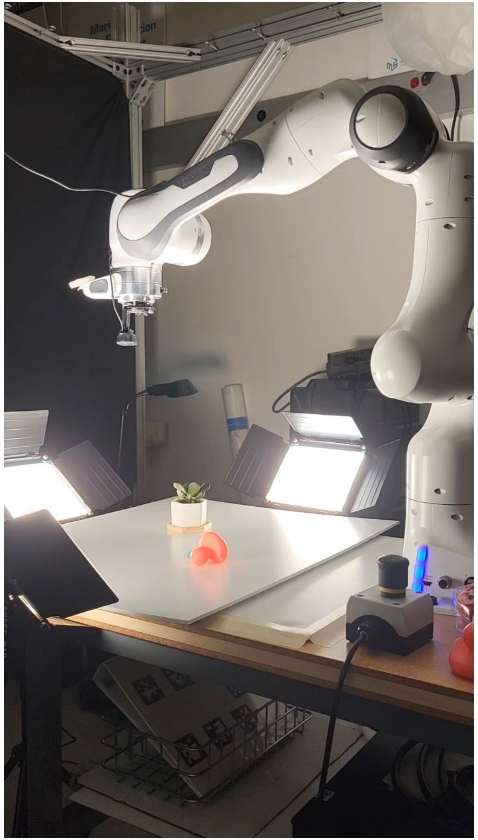
# Unsupervised Physical Learning

Robots create perception pipelines through automated data collection, and ergodic control **guides** sensory experience without supervision.




The robot's motion is ergodic with respect to the spatially distributed entropy

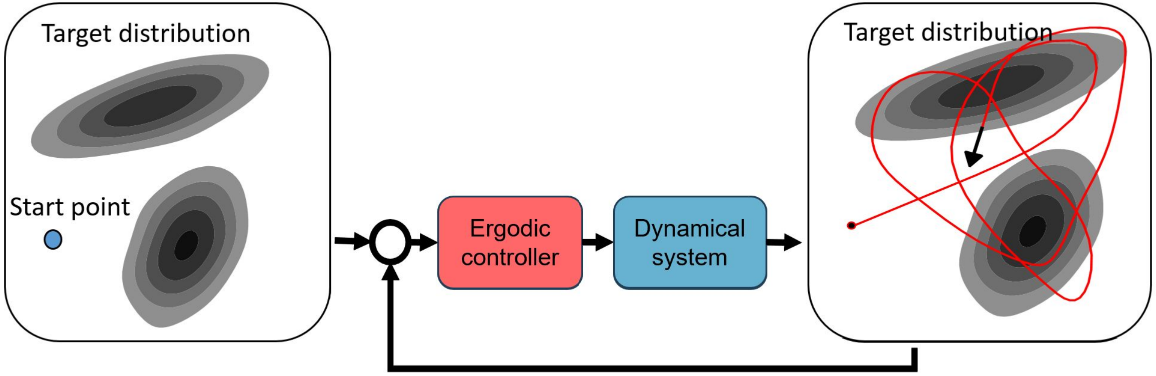


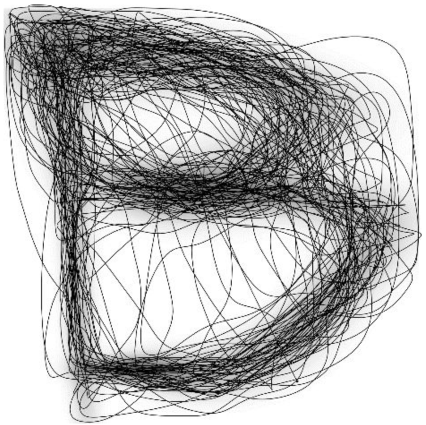



# Sylvain Calinon’s Ergodic Pitch

# Ergodic control: Why is it exciting ?

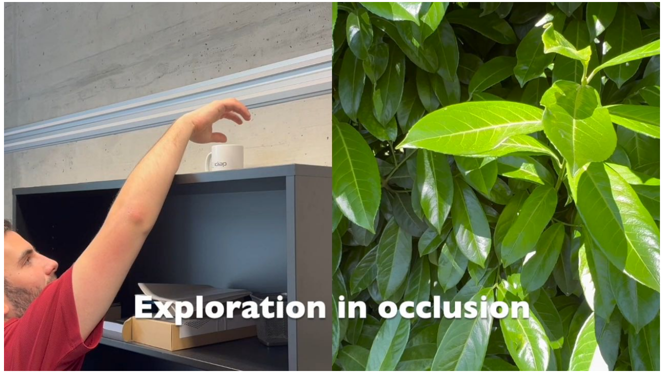
**Generality of the formulation:** Tracking as special case of ergodic control



**Natural exploration**
without any randomness!

# Ergodic control: Why is it exciting ?




**Ergodicity**: Difference between the time-averaged spatial statistics of the agent's trajectory and the target distribution to search in

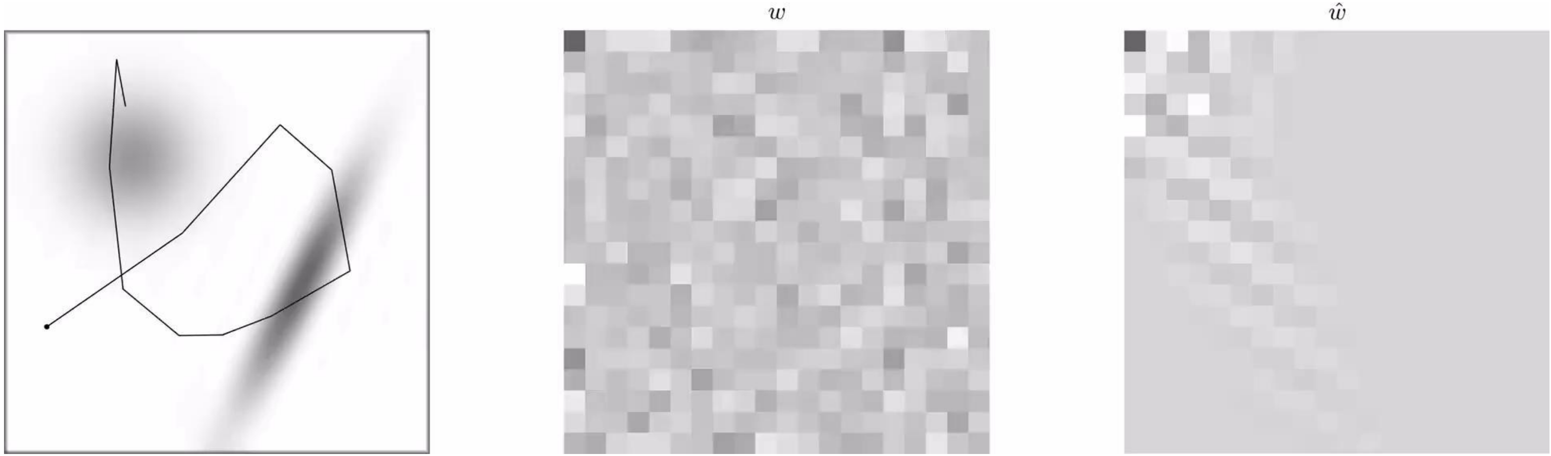
**→ Simple yet powerful principle!**

*Input:* Spatial distribution
*Output:* Control commands

*Simple cost:*
Matching Fourier series coefficients

fixed weights

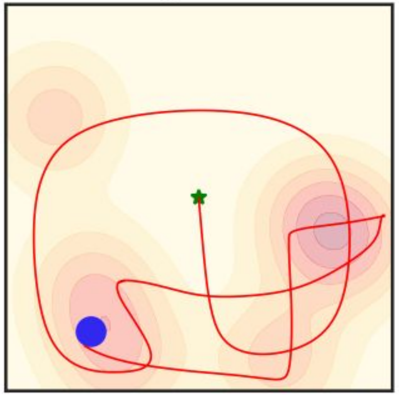
$$\min_{u(t)} \sum_{k\in\mathcal{K}} \Lambda_k (w_k - \hat{w}_k)^2$$



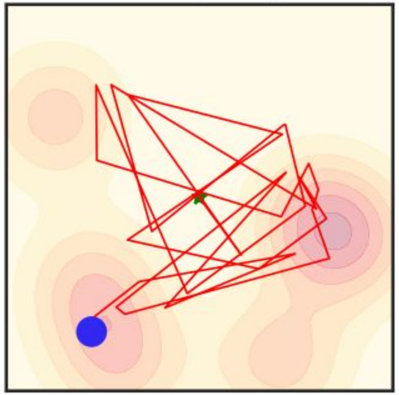
# Ergodic control: Why is it exciting ?

- **Deep fundamental challenges** linking machine learning, optimal control, signal processing and information theory
- **Achieves manipulation tasks robustly**, by not only relying on accurate sensors, but instead using a control strategy to cope with limited or inaccurate sensing information
- Different from stochastic or patterned search! → Provides **a natural way of searching**

Ergodic search

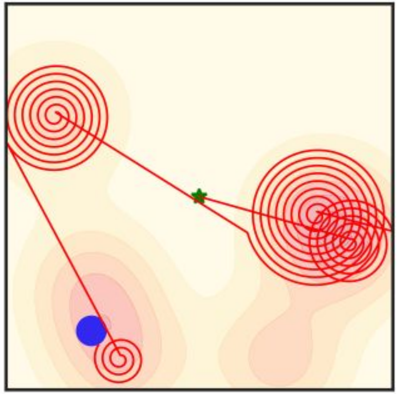


Stochastic search



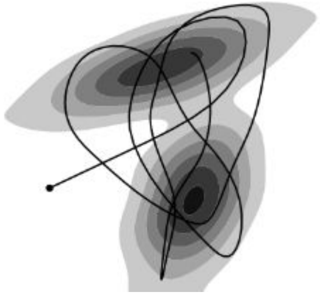
→ does not take into account motion cost!

Patterned search

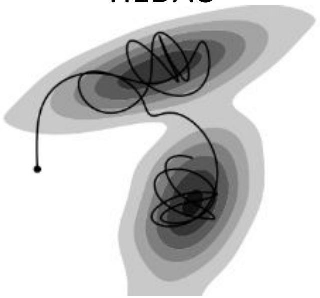


→ optimal only when duration is known!

SMC



HEDAC



# Guillaume’s Ergodic Pitch

Guillaume Sartoretti

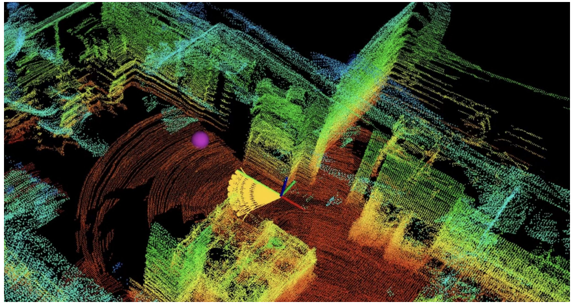
Assistant Professor, National University of Singapore

http://www.marmotlab.org

# Informative Path Planning (IPP)


Monitoring

Reconstruction

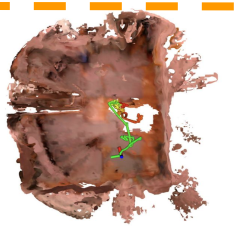


Exploration

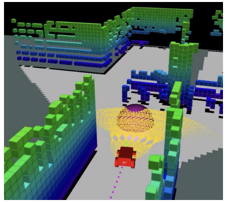
Lidar

Camera

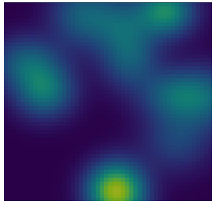
Thermal sensor



TSDF



Occupancy grid



Gaussian distribution

Popović, Marija, et al. "An informative path planning framework for UAV-based terrain monitoring." *Autonomous Robots* 44.6 (2020): 889-911.
Schmid, Lukas, et al. "An efficient sampling-based method for online informative path planning in unknown environments." *IEEE Robotics and Automation Letters* 5.2 (2020): 1500-1507.
Cao, Yuhong, et al. "Deep Reinforcement Learning-based Large-scale Robot Exploration." *IEEE Robotics and Automation Letters* (2024).

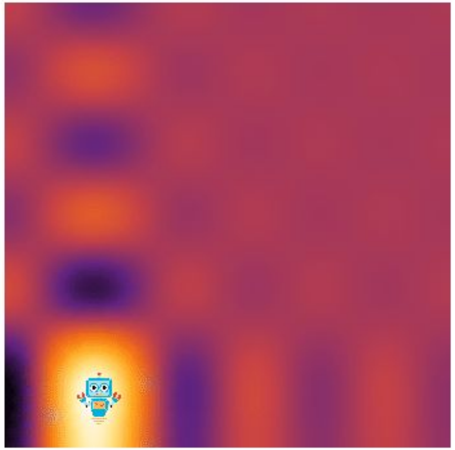
# Single-Agent Ergodic Coverage

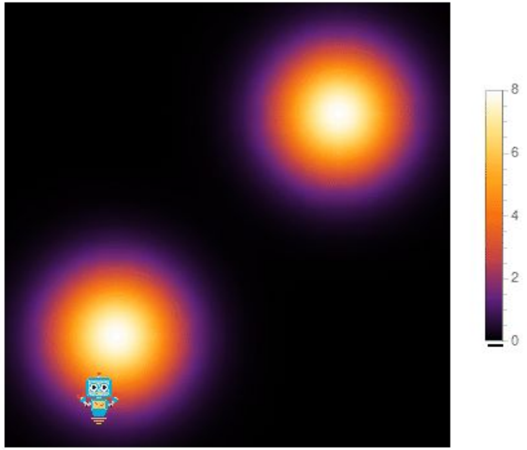
$$\boldsymbol{\Phi}(\gamma, \xi) = \sum_{k=0}^{m} \alpha_k \left(c_k\big(\gamma(t)\big) - \xi_k\right)^2$$

Find **controls** that **minimize** the ergodic metric subject to **dynamic constraints**

$$\boldsymbol{u}^* = \arg\min_u \boldsymbol{\Phi}(\gamma, \xi)$$

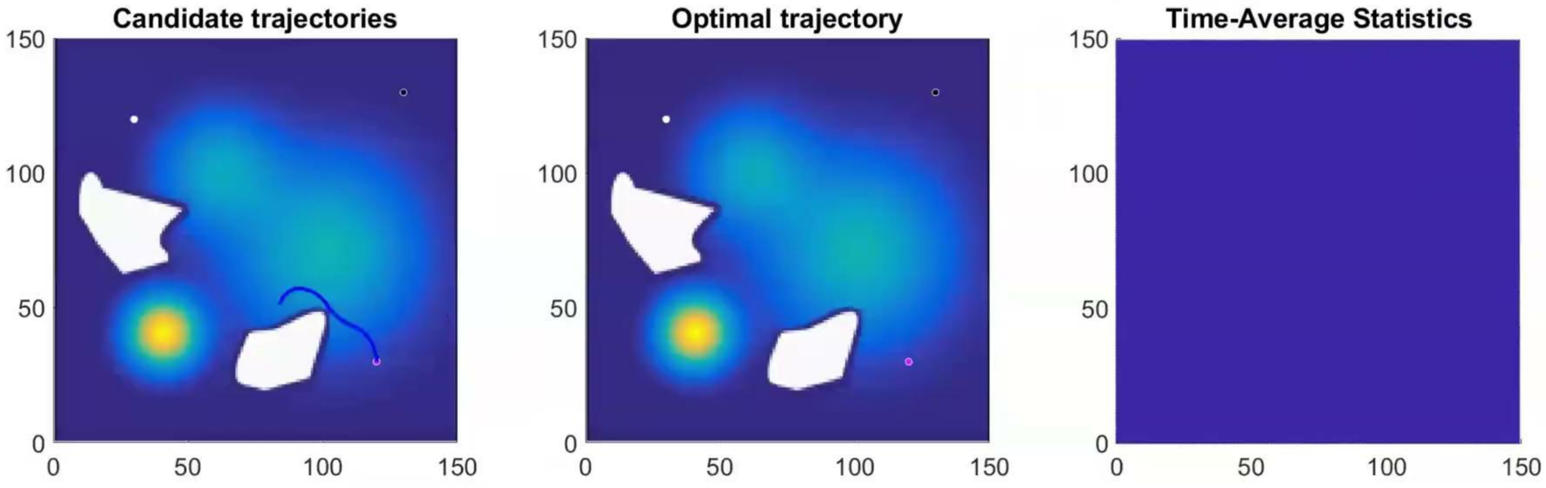
$$\text{subject to } \dot{\boldsymbol{q}} = f(\boldsymbol{q}(t), \boldsymbol{u}(t))$$



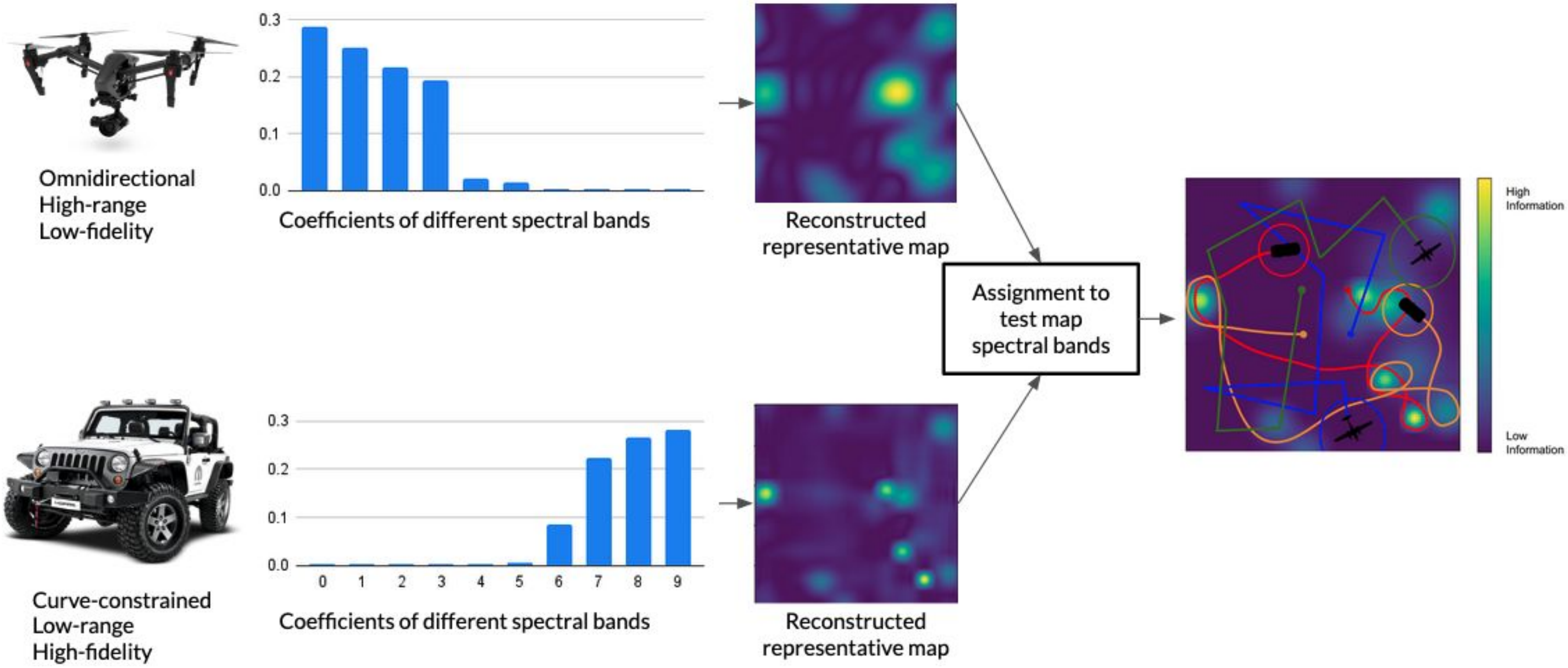


# Multi-Agent Ergodic Coverage

$$\boldsymbol{\Phi}(\gamma, \xi) = \sum_{k=0}^{m} \alpha_k \left(c_k\big(\gamma(t)\big) - \xi_k\right)^2$$



# Heterogeneous Multi-Agent Coverage

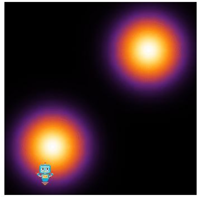


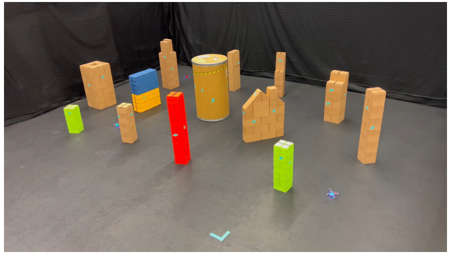
# A Path Towards Deterministic and Intentional Robotic Exploration

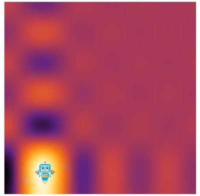
Ian Abraham,
Asst. Prof Mechanical Engineering and Computer Science,
Yale University

Yale SCHOOL OF ENGINEERING & APPLIED SCIENCE

# Why Ergodic Control is Exciting!

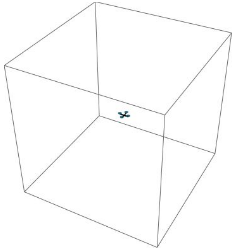


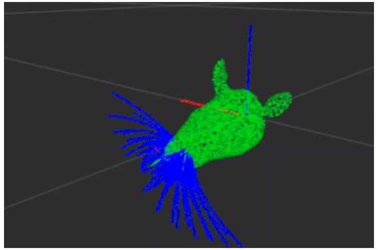


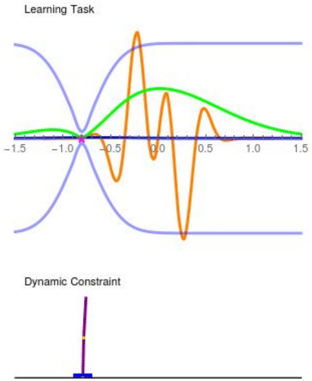


***<u>Ergodic Exploration/Search</u>***

- Deterministic Exploratory Behaviors
- Nonlinear/nonconvex problem with many “good” local minima
  - Many solutions to the same problem
- Independent of robot dynamics/spatial scale/sensor/information measure
  - We can analyze exploration from a more general perspective
- Non-Myopic Exploration (formulated over long planning horizons)



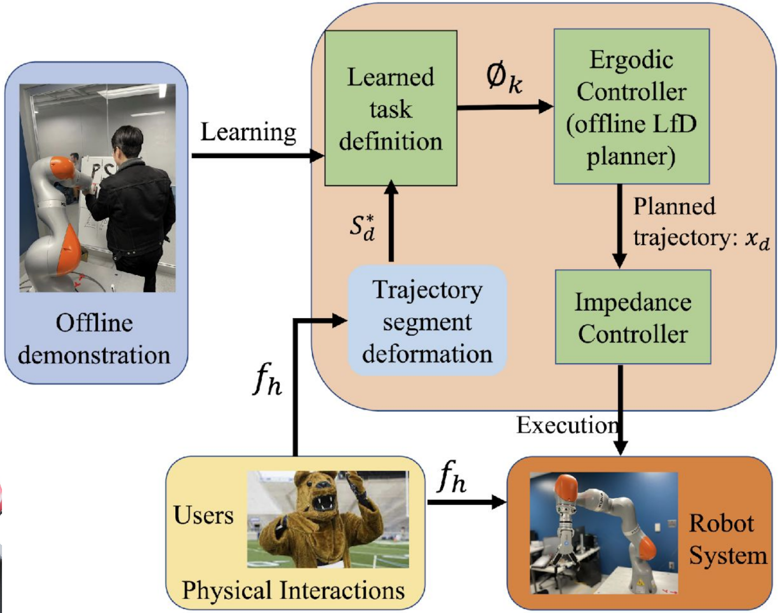


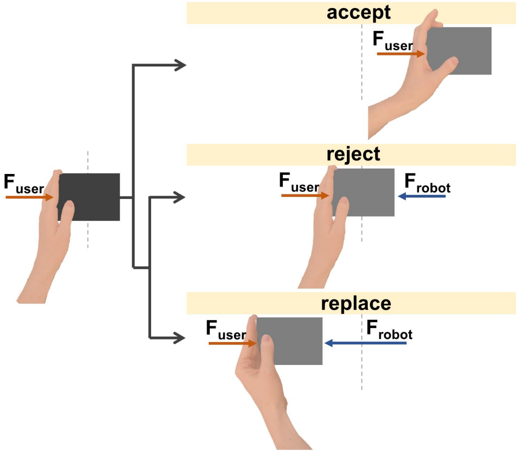



[ICRA 2024] Seewald, et al. "Energy-Aware Ergodic Search: Continuous Exploration for Multi-Agent Systems with Battery Constraints."
[RSS 2023] Dong, et. al. “Time-Optimal Ergodic Search”
[ICRA 2023] Lerch, et. al.  "Safety-critical ergodic exploration in cluttered environments via control barrier functions
[TASE 2023] Abraham , et. al. "An ergodic measure for active learning from equilibrium."

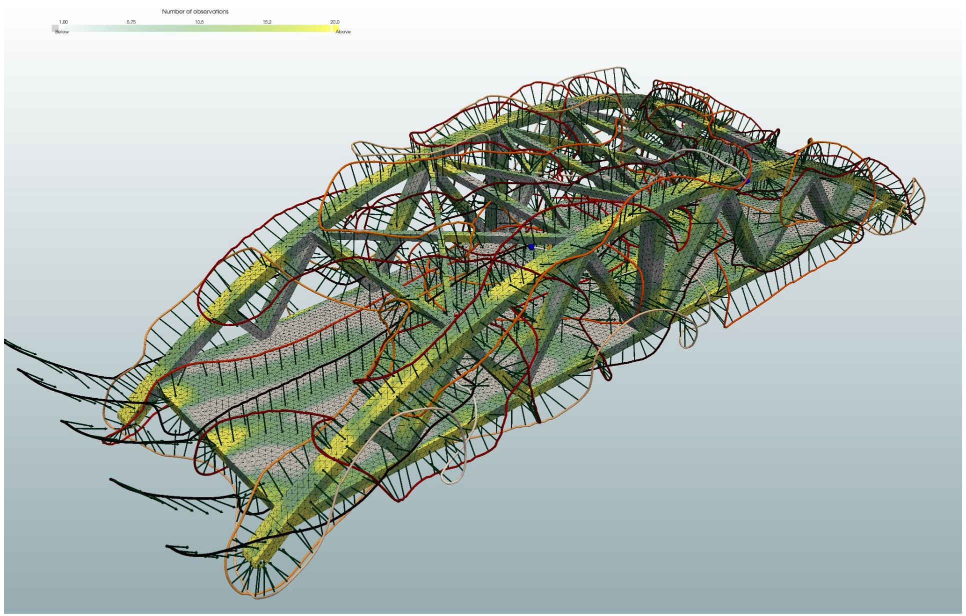
# Ergodic Control facilitates Human Robot Interaction

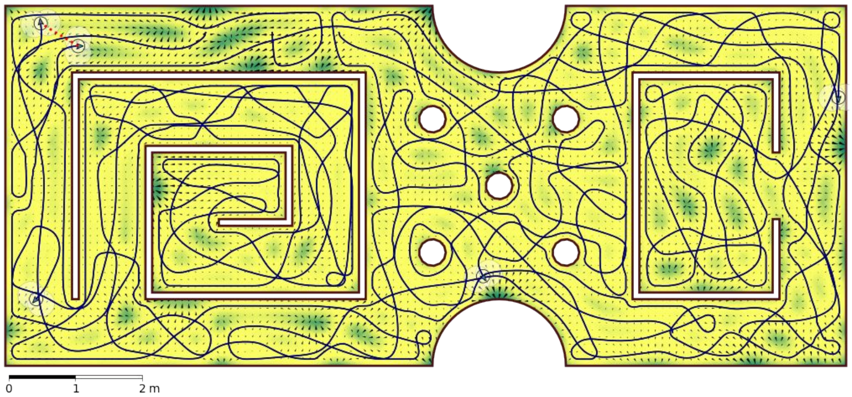
- Rehabilitation Robotics
- Virtual Training & Haptics
- Learning From Demonstration

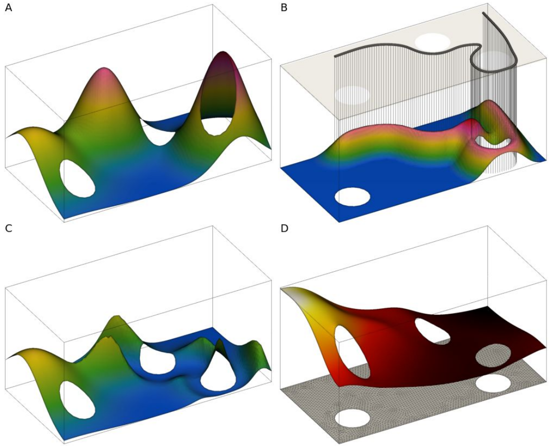



# Stefan's pitch: Ergodic control driven by the **heat equation**




Applications: UAV search, surveying, spraying

# Session 2 (10:30am-12pm)