Sampling-Based Planning for Single-/Multi-Robot Ergodic Coverage

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ICRA 2024 Tutorial on Ergodic Control, May 13th

http://www.marmotlab.org

Outline

- Informative Path Planning
    - Definition, important considerations
- Ergodic Coverage
    - Spectral-based constrained optimization
- Sampling-Based Planning
    - Stochastic Optimization, Cross-Entropy Planning

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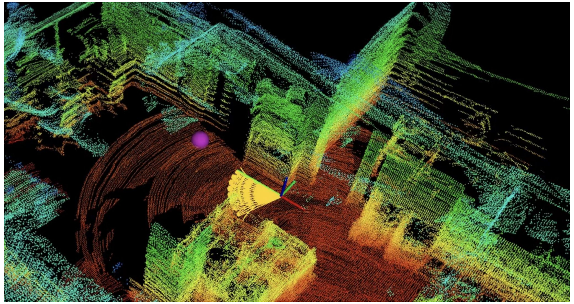
Informative Path Planning (IPP)

Monitoring

Reconstruction

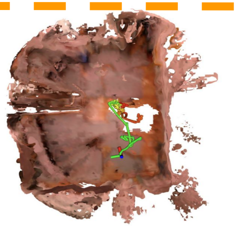


Exploration

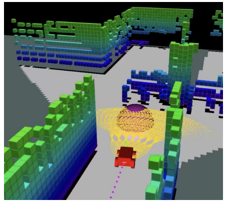
Lidar

Camera

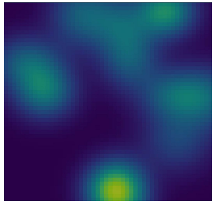
Thermal sensor



TSDF



Occupancy grid



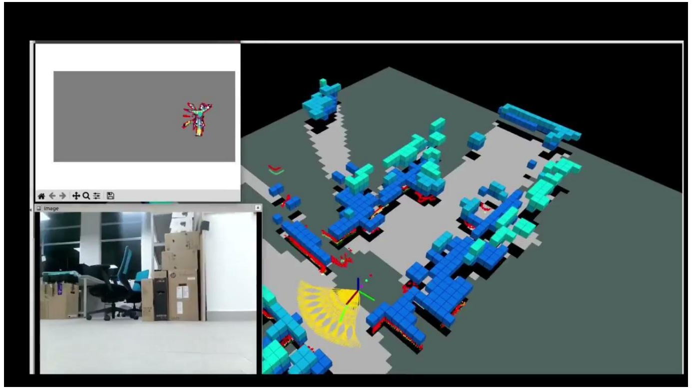
Gaussian distribution

Popović, Marija, et al. "An informative path planning framework for UAV-based terrain monitoring." *Autonomous Robots* 44.6 (2020): 889-911.
Schmid, Lukas, et al. "An efficient sampling-based method for online informative path planning in unknown environments." *IEEE Robotics and Automation Letters* 5.2 (2020): 1500-1507.
Cao, Yuhong, et al. "Deep Reinforcement Learning-based Large-scale Robot Exploration." *IEEE Robotics and Automation Letters* (2024).

Informative Path Planning (IPP)

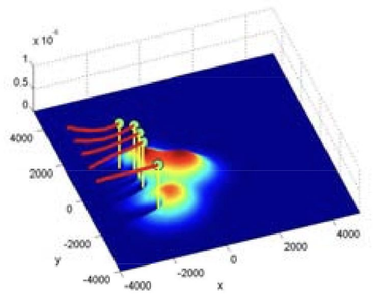
Goal: plan the path of a mobile robot, to efficiently gather sensor measurements in a known/unknown environment and reconstruct some underlying distribution of interest.

- ***Non-adaptive*** IPP: the environment is known → one-shot plan.
- ***Adaptive*** IPP: the environment is unknown or partially known.

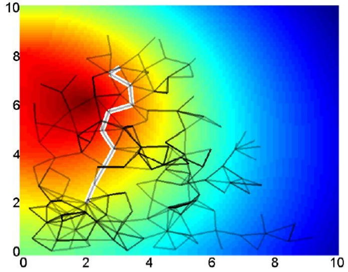


Cao, Yuhong, et al. "CAtNIPP: Context-aware attention-based network for informative path planning." *Conference on Robot Learning*. PMLR, 2023.
Cao, Yuhong, et al. "Deep Reinforcement Learning-based Large-scale Robot Exploration." *IEEE Robotics and Automation Letters* (2024).

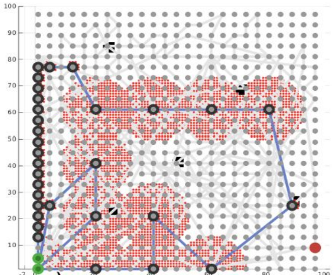
Existing Methods



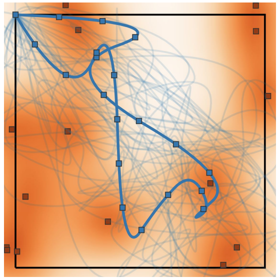
Greedy



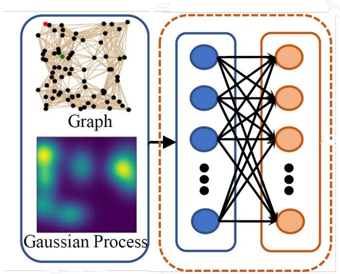
RRT-based



TSP-based



Meta heuristic-based



Learning-based

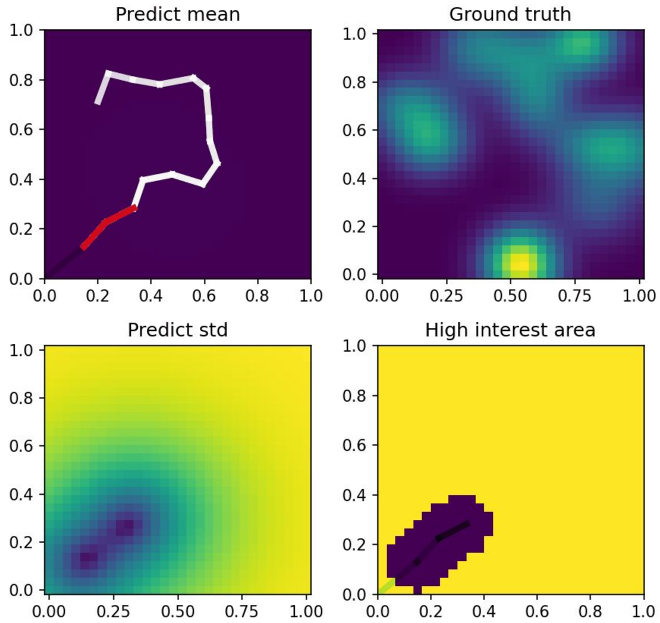
Wong, El-Mane, Frédéric Bourgault, and Tomonari Furukawa. "Multi-vehicle Bayesian search for multiple lost targets." *Proceedings of the 2005 ieee international conference on robotics and automation*. IEEE, 2005.
Hollinger, Geoffrey A., and Gaurav S. Sukhatme. "Sampling-based robotic information gathering algorithms." *The International Journal of Robotics Research* 33.9 (2014): 1271-1287.
Arora, Sankalp, and Sebastian Scherer. "Randomized algorithm for informative path planning with budget constraints." *2017 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2017.
Hitz, Gregory, et al. "Adaptive continuous-space informative path planning for online environmental monitoring." *Journal of Field Robotics* 34.8 (2017): 1427-1449.

Measurements and Agent Map/Belief

2D Gaussian Process as agent belief

- Represent a ***continuous distribution*** by interpolating between ***discrete measurements***.
- Provide a measure of ***uncertainty*** to assess the accuracy of interpolations.
- Model sensor capacity through the ***kernel*** function.

$$\mu = \mu(\mathcal{X}^*) + K(\mathcal{X}^*, \mathcal{X})[K(\mathcal{X}, \mathcal{X}) + \sigma_n^2 I]^{-1}(\mathcal{Y} - \mu(\mathcal{X}))$$
$$P = K(\mathcal{X}^*, \mathcal{X}^*) - K(\mathcal{X}^*, \mathcal{X})[K(\mathcal{X}, \mathcal{X}) + \sigma_n^2 I]^{-1} \times K(\mathcal{X}^*, \mathcal{X})^T$$



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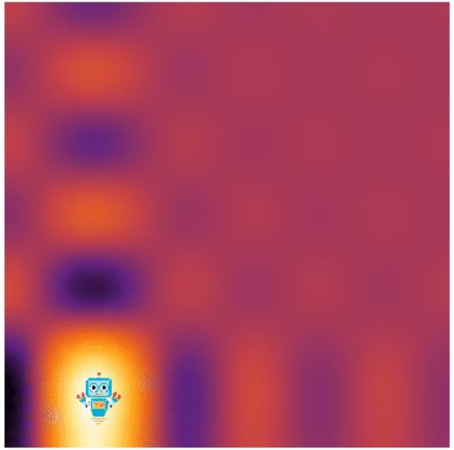
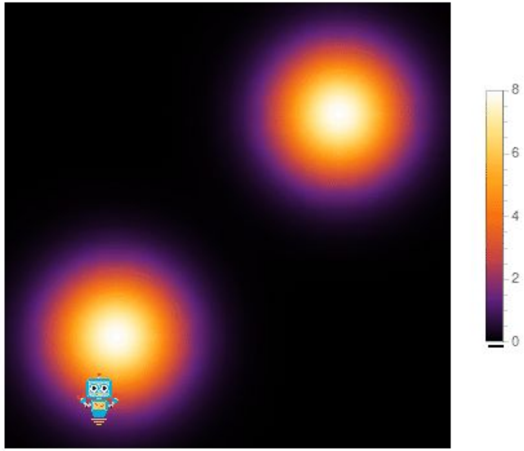
Trajectory Optimization for Ergodic Coverage

Find **controls**

$$\boxed{\boldsymbol{u}^*} = \arg\min_u \boldsymbol{\Phi}(\gamma, \xi)$$

$$\text{subject to } \dot{\boldsymbol{q}} = f(\boldsymbol{q}(t), \boldsymbol{u}(t))$$





[1] Mathew, G., Mezìc, I.: Metrics for ergodicity and design of ergodic dynamics for multi-agent systems. Physica D: Nonlinear Phenomena 240(4), 432–442 (2011)

Trajectory Optimization for Ergodic Coverage

Find **controls** that **minimize** the ergodic metric

$$\boldsymbol{u}^* = \arg\min_u \boldsymbol{\Phi}(\gamma, \xi)$$

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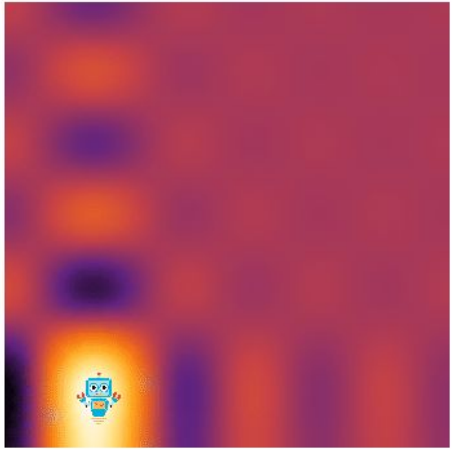
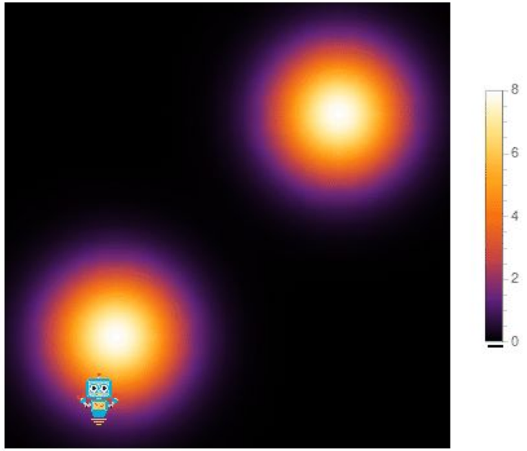




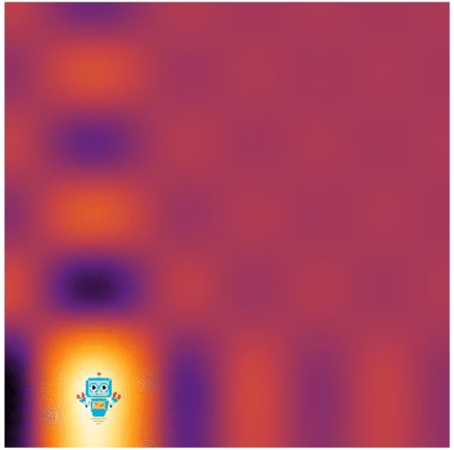
[1] Mathew, G., Mezìc, I.: Metrics for ergodicity and design of ergodic dynamics for multi-agent systems. Physica D: Nonlinear Phenomena 240(4), 432–442 (2011)

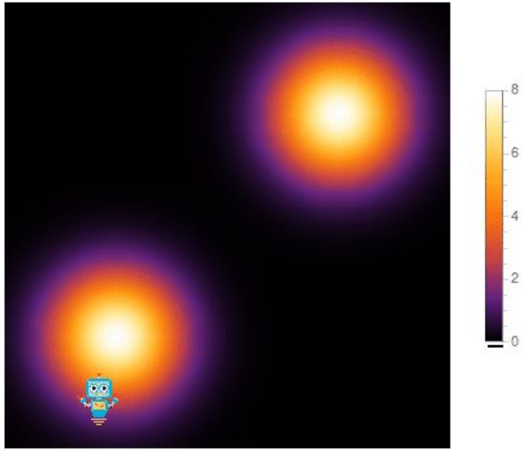
Trajectory Optimization for Ergodic Coverage

Find **controls** that
minimize the ergodic metric
subject to **dynamic constraints**

$$\boldsymbol{u}^* = \arg\min_u \boldsymbol{\Phi}(\gamma, \xi)$$

$$\text{subject to } \boxed{\dot{\boldsymbol{q}} = f(\boldsymbol{q}(t), \boldsymbol{u}(t))}$$





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Background on Ergodic Trajectory Optimization

We seek a long-term path that minimizes the **ergodic metric**

$$\boldsymbol{\Phi}(\gamma, \xi) = \sum_{k=0}^{m} \alpha_k \left(c_k\big(\gamma(t)\big) - \xi_k\right)^2$$

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Background on Ergodic Trajectory Optimization

We seek a long-term path γ that minimizes the **ergodic metric**

$$\boldsymbol{\Phi}(\gamma, \xi) = \sum_{k=0}^{m} \alpha_k (c_k(\gamma(t)) - \xi_k)^2$$

$$\gamma = [x_0, \dots, x_{T-1}]$$

Robot trajectory

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Robot trajectory

$$\xi(x)$$

Utility function

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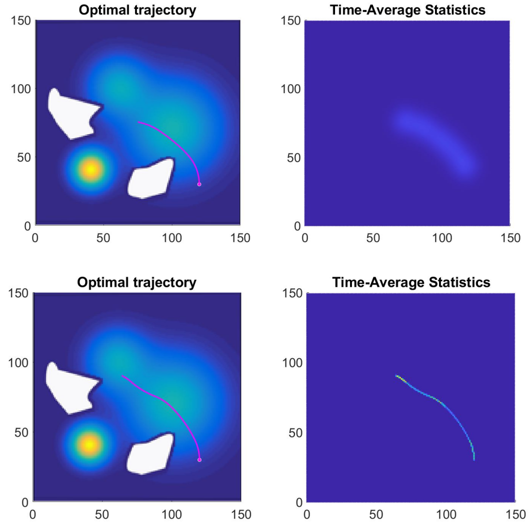
Robot trajectory

$$\xi(x)$$

Utility function

$$c_k(\gamma(t)) = \frac{1}{T} \sum_{t=0}^{T-1} F_k(x_t)$$

Time-averaged trajectory statistics



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Robot trajectory

$$\xi(x)$$

Utility function

$$c_k(\gamma(t)) = \frac{1}{T} \sum_{t=0}^{T-1} F_k(x_t)$$

Time-averaged trajectory statistics

$$\xi_k = \int_X F_k(x) \xi(x) dx$$

Utility distribution

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Background on Ergodic Trajectory Optimization

We seek γ that minimizes the **ergodic metric**

$$\boldsymbol{\Phi}(\gamma,\xi) = \sum_{k=0}^{m} \alpha_k \left(c_k(\gamma(t)) - \xi_k\right)^2$$

$m = 100$

Number of Fourier coefficients

$\gamma = [x_0, \dots, x_{T-1}]$

Robot trajectory

$\xi(x)$

Utility function

$$c_k(\gamma(t)) = \frac{1}{T} \sum_{t=0}^{T-1} F_k(x_t)$$

Time-averaged trajectory statistics

$$\xi_k = \int_X F_k(x)\xi(x)dx$$

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$m = 100$

Number of Fourier coefficients

$$\alpha_k = \sqrt{(1+k^2)^{-(d+1)}}$$

normalizing coefficients

$$\gamma = [x_0, \dots, x_{T-1}]$$

Robot trajectory

$$\xi(x)$$

Utility function

$$c_k(\gamma(t)) = \frac{1}{T}\sum_{t=0}^{T-1} F_k(x_t)$$

Time-averaged trajectory statistics

$$\xi_k = \int_X F_k(x)\xi(x)dx$$

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Dynamics-Aware Optimization

For simple agent dynamics, the constrained optimization system below can be exactly solved to obtain optimal controls (e.g., [2]):

$$\boldsymbol{u}^* = \arg\min_u \boldsymbol{\Phi}(\gamma, \xi)$$

$$\text{subject to } \dot{\boldsymbol{q}} = f(\boldsymbol{q}(t), \boldsymbol{u}(t))$$

However, for general cases, and at lower computational cost:

<u>Stochastic Trajectory Optimization</u>

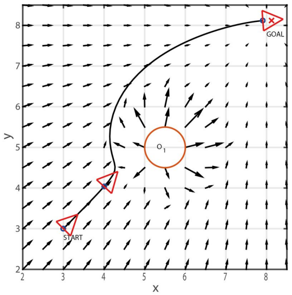
[2] Miller, L. M., & Murphey, T. D. (2013). Trajectory optimization for continuous ergodic exploration. In 2013 American Control Conference (pp. 4196-4201).

Stochastic Trajectory Optimization

Deterministic vs Stochastic trajectory optimization

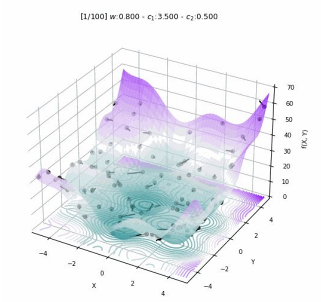
- DTO: Artificial Potential Field, A*, etc.
- STO: Rapidly exploring random tree, Particle swarm optimization,

Simulated annealing, Bayesian optimization, etc.



APF [4]

RRT [5]



PSO[6]

We will focus on sampling-based, cross-entropy planning [3]

[3] Kobilarov, M. (2012). Cross-Entropy Randomized Motion Planning. Robotics: Science and Systems VII, 153.
[4] Fedele, G. (2018). Obstacles avoidance based on switching potential functions. Journal of Intelligent & Robotic Systems, 90, pp.387-405.
[5] LaValle, Steven M. (1998). Rapidly-exploring random trees: A new tool for path planning. Technical Report (TR 98–11).
[6] Axel Thevenot. https://towardsdatascience.com/particle-swarm-optimization-visually-explained-46289eeb2e14

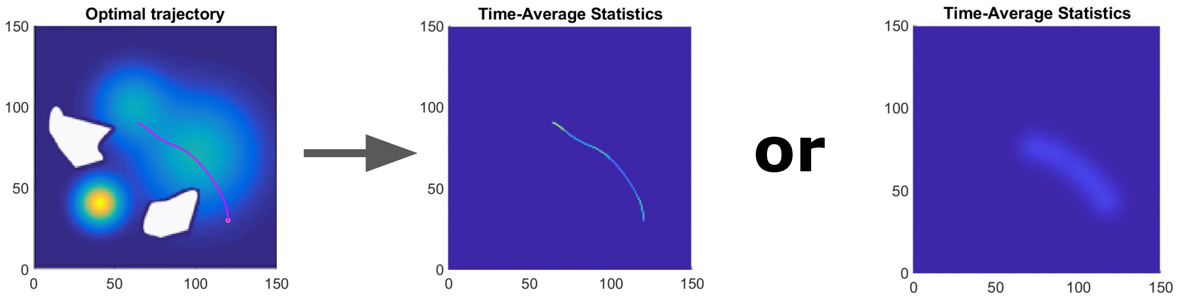
Sampling-based Cross-Entropy Planning

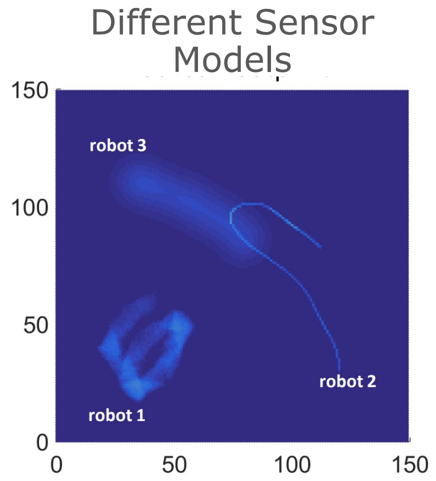
Formulation

- Sample a vector z from a Gaussian mixture model p(z;v)

$$p(z;v) = \sum_{k=1}^{K} \frac{w_k}{\sqrt{(2\pi)^{n_z} |\Sigma_k|}} e^{-\frac{1}{2}(z-\mu_k)^T \sigma_k^{-1}(z-\mu_k)}$$

- Sample a set of trajectories, and evaluate the cost function J(z) (here, the Ergodic metric).





[3] Kobilarov, M. (2012). Cross-Entropy Randomized Motion Planning. Robotics: Science and Systems VII, 153.

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- Sample a set of trajectories, and evaluate the cost function J(z) (here, the Ergodic metric).

- use a subset of elite trajectories (e.g., best 20%) and update v

- after some iterations, p(z;v) tends to a delta distribution, yielding (near-)optimal paths wrt Ergodicity.

[3] Kobilarov, M. (2012). Cross-Entropy Randomized Motion Planning. Robotics: Science and Systems VII, 153.

Example: Ergodic Coverage by UGVs

Dubins car model with state q = (x, y, θ) of coordinates and orientation

$$\dot{x} = ucos\theta, \quad \dot{y} = usin\theta, \quad \dot{\theta} = v$$

Define primitives based on forward velocity v and turning rate w (based on the agent’s dynamics)

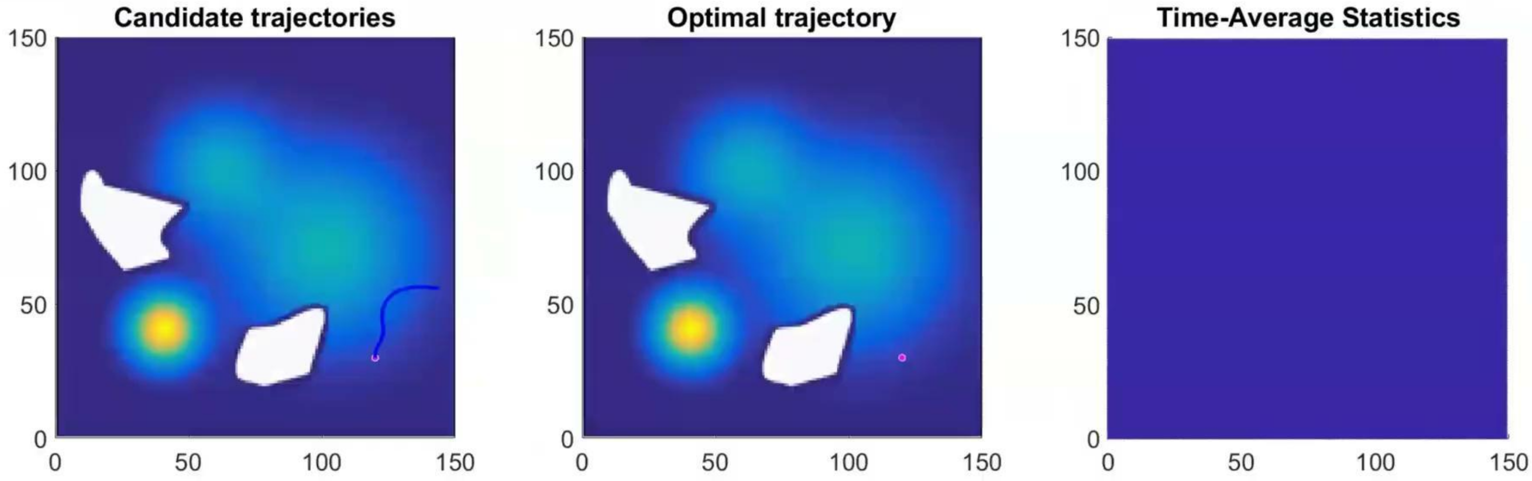
- straight lines (constant velocity v, and w = 0)
- arcs of radius v/w (v,w ≠ 0)

Importance sampling and evaluation

- Path: sequences of primitives (z = [v1, w1, v2, w2, …, vn, wn])
- calculate ergodicity and update Gaussian mixture model
- iterate above steps until convergence/fixed # of iterations

[3] Kobilarov, M. (2012). Cross-Entropy Randomized Motion Planning. Robotics: Science and Systems VII, 153.

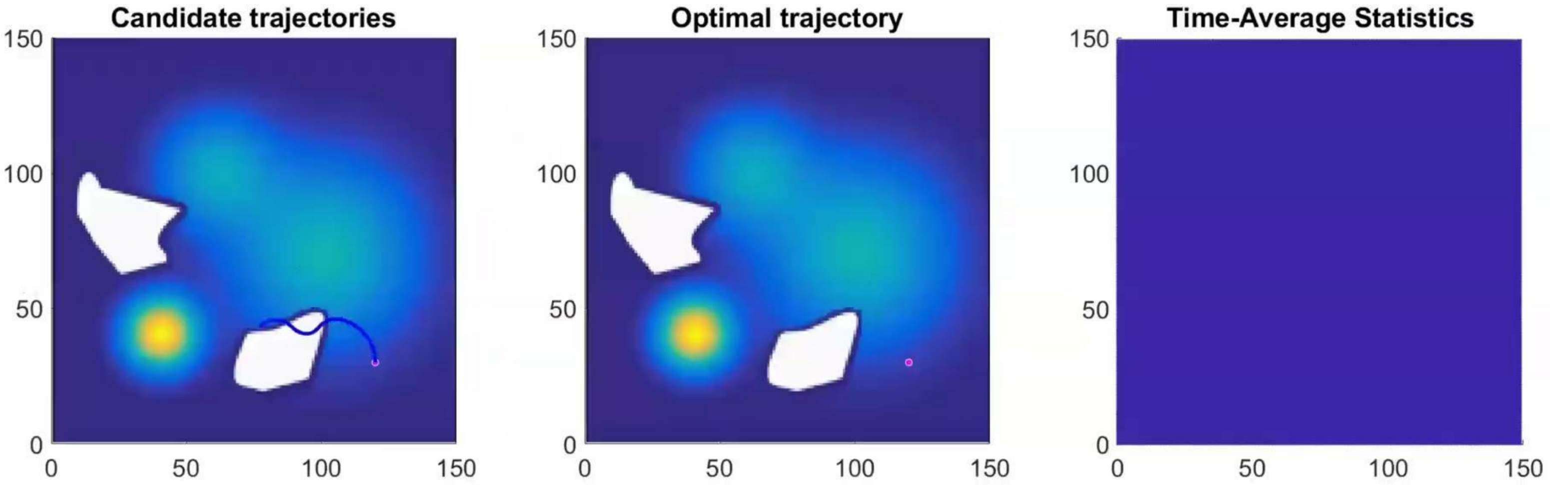
Sampling-based Cross-Entropy Ergodic Planning



[7] https://github.com/biorobotics/stoec

[8] Ayvali, E., Salman, H., & Choset, H. (2017, September). Ergodic coverage in constrained environments using stochastic trajectory optimization. *IROS 2017,* (pp. 5204-5210).

Sampling-based Cross-Entropy Ergodic Planning



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Summary

1. Single-agent coverage can greatly benefit from Ergodicity when a prior over the domain is available:

    - Avoids myopic decision-making, by naturally balancing exploration and exploitation in spectral domain (*long-term*, *large-scale*).

2. More complex agent dynamics can be handled via stochastic trajectory optimization (e.g., sampling-based methods):

    - Linear cost in number of sample.
    - Sub-/Near-optimal results.