

Applications: Search with UAVs

Tutorial at ICRA 2024, Yokohma

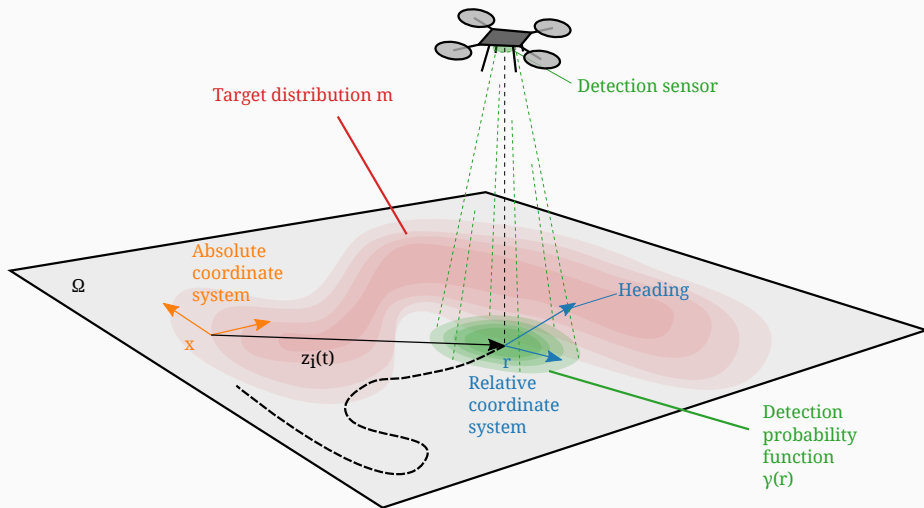
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Two-dimensional search problem [Ivi20]



Non-radial sensing functions

The use of non-radial sensing functions requires consideration of agent rotation/orientation [IAD19] [Ivi20]

[IAD19] S. Ivić, A. Andrejčuk, and S. Družeta. “Autonomous control for multi-agent non-uniform spraying”. In: *Applied Soft Computing* 80 (2019), pp. 742–760

[Ivi20] S. Ivić. “Motion Control for Autonomous Heterogeneous Multiagent Area Search in Uncertain Conditions”. In: *IEEE Transactions on Cybernetics* (2020)

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$$c(\mathbf{x}, t) = \frac{1}{Nt} \sum_{i=1}^N \int_0^t \gamma(\mathbf{R}(\theta_i(\tau)) \cdot \mathbf{x} - \mathbf{z}_i(\tau)) d\tau$$

where

- θ is the angle of agent's direction
- \mathbf{R} is the rotation matrix

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

- $\gamma(\mathbf{r})$ is sensing function in relative coordinates

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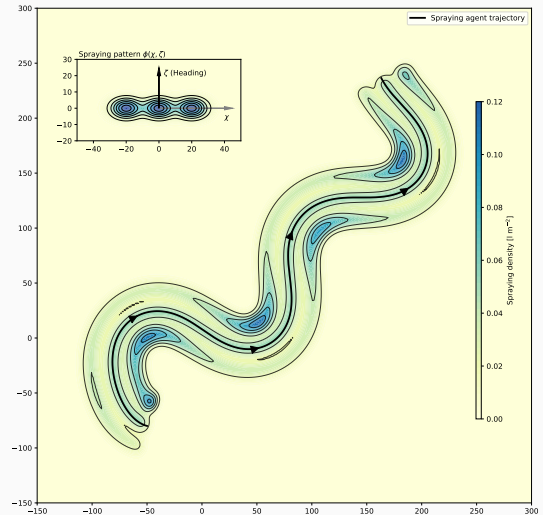
$$c(\mathbf{x}, t) = \frac{1}{Nt} \sum_{i=1}^N \int_0^t \gamma(\mathbf{R}(\theta_i(\tau)) \cdot \mathbf{x} - \mathbf{z}_i(\tau)) d\tau$$

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Using non-radial basis functions allows more realistic applications:

- UAV spraying [IAD19] 
- UAV altitude control [LJ124] 



[IAD19] S. Ivić, A. Andrejčuk, and S. Družeta. “Autonomous control for multi-agent non-uniform spraying”. In: *Applied Soft Computing* 80 (2019), pp. 742–760

[LJ124] L. Lanča, K. Jakac, and S. Ivić. “Model predictive altitude and velocity control in ergodic potential field directed multi-UAV search”. In: *arXiv preprint arXiv:2401.02899* (2024)

The source in the heat equation can be modeled differently!
It needs to (somehow) represent the field which we want to be **minimized**

[Koo56] B.O. Koopman. "The theory of search. II. Target detection". In: *Operations research* 4.5 (1956), pp. 503–531

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According to [Koo56], when the search is performed continuously until time t , the probability of target detection $p(t)$ is given by

$$p(t) = 1 - e^{-\gamma t}, \quad t \geq 0.$$

The remaining target probability distribution with continuous (sensing) action c in [Ivi20]:

$$m(\mathbf{x}, t) = m_0(\mathbf{x}) \cdot e^{-c(\mathbf{x}, t)}.$$

UAV search (heterogeneous agents) [Ivi20] 

[Koo56] B.O. Koopman. "The theory of search. II. Target detection". In: *Operations research* 4,5 (1956), pp. 503–531

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Solving the heat PDE with Finite Element Method (FEM) [\[ISC21\]](#):

- Irregularly shaped domains
- Including static obstacles in elegant and robust manner intrinsic to HEDAC's fundamental idea
- Gradient comes "free"
- Interpolation of potential and its gradient comes "free"

Setting up and solving heat equation in FEM (ngsolve)

```
mesh = netgen.read_gsmh.ReadGmsh('case_01_simple.msh')
fe_mesh = ngsolve.Mesh(mesh)
fes = ngsolve.H1(fe_mesh, order=2)

# Define trial- and test-functions
u = fes.TrialFunction()
v = fes.TestFunction()

m0 = ngsolve.GridFunction(fes) # Shoul put some values in m0
c = ngsolve.GridFunction(fes) # Coverage needs to be updated in each step
m.Set(m0 * ngsolve.exp(-self.c))

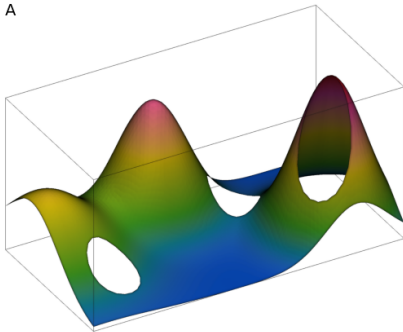
# The bilinear-form
a = ngsolve.BilinearForm(self.fes)
a += ngsolve.SymbolicBFI(alpha * ngsolve.grad(u) * ngsolve.grad(self.v) +
                        beta * u * v)
a.Assemble()
# Precompute the inverse:
a_inv = a.mat.Inverse(fes.FreeDofs())

# Set the right-hand side of the heat equation
f = ngsolve.LinearForm(self.fes)
f += ngsolve.SymbolicLFI(m / numpy.average(m.vec[:nn]) * v)
f.Assemble()

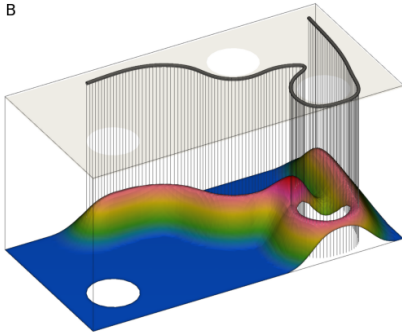
# The solution field u
u = GridFunction(fes)
u.vec.data = a_inv * f.vec
```

```
# Getting the gradient of u  
grad_u = ngsolve.grad(u)  
  
# Agent position (xa, ya)  
pos = fe_mesh(xa, ya)  
  
# Get the gradient at pos  
vxy = grad_u(pos)
```

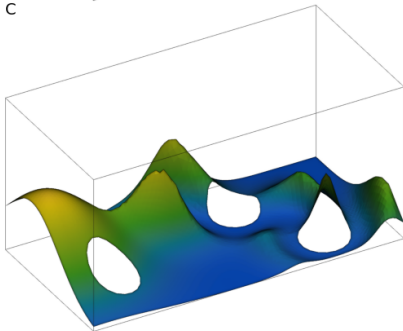
A



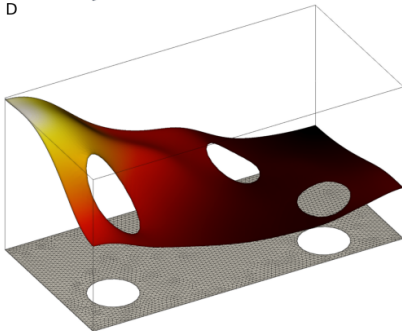
B

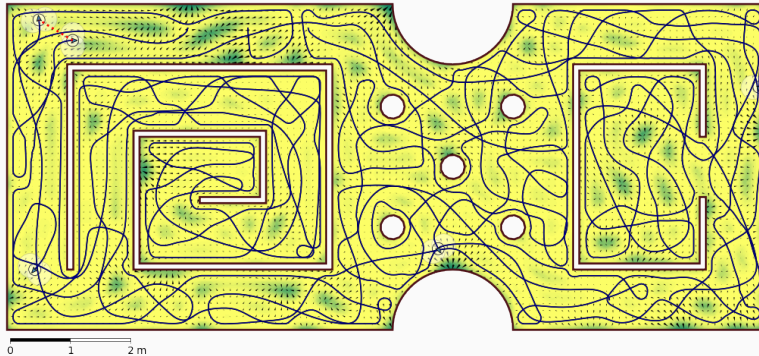


C






D

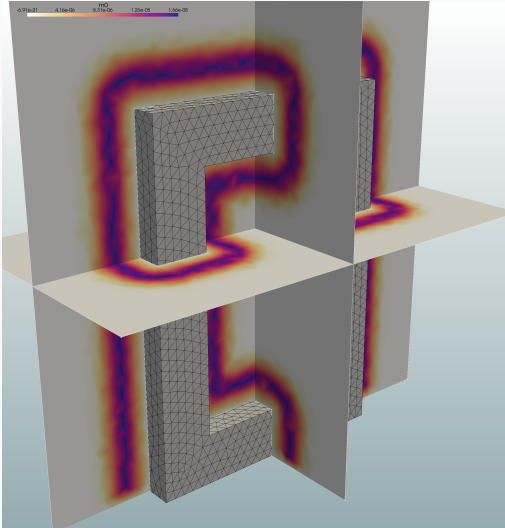




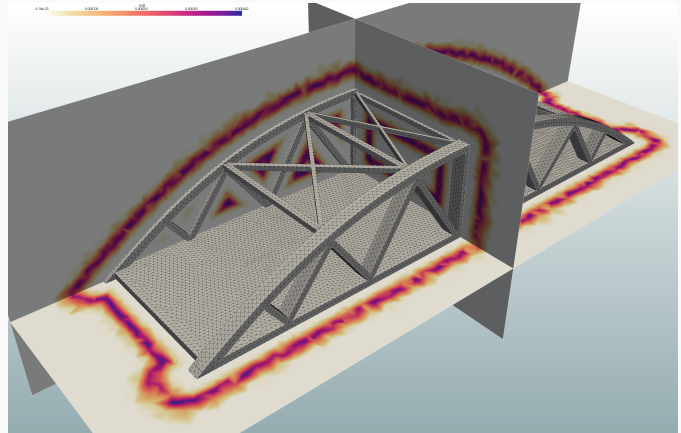
Animations (for simulations in [\[ISC21\]](#)):

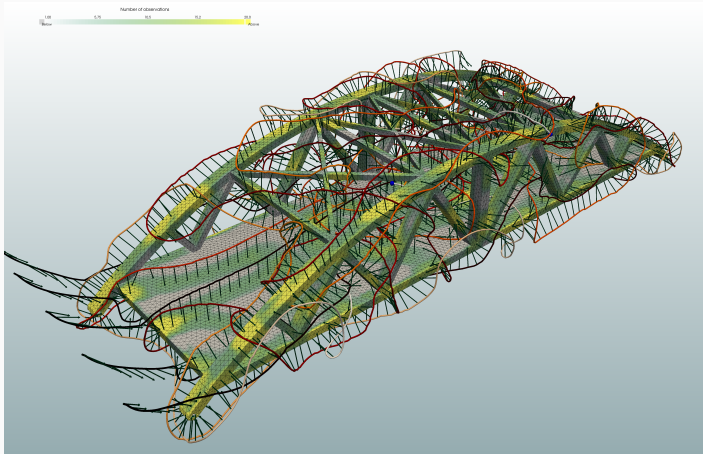
- Simple case 
- Governors Island 
- Stockholm archipelago 

[\[ISC21\]](#) S. Ivić, A. Sikirica, and B. Crnković. “Constrained multi-agent ergodic area surveying control based on finite element approximation of the potential field”. In: *arXiv preprint arXiv:2109.10756* (2021)



Finite elements mesh allows to model complex geometries
[Ivi+23]

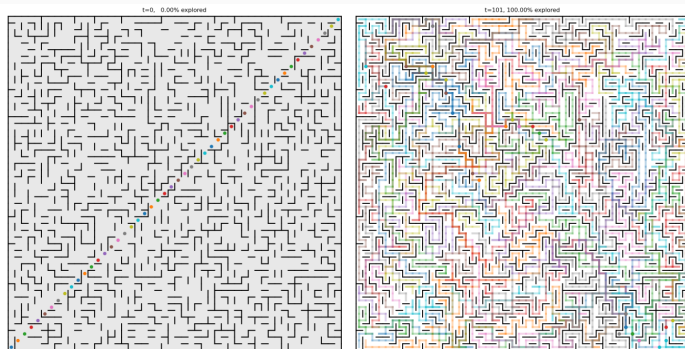
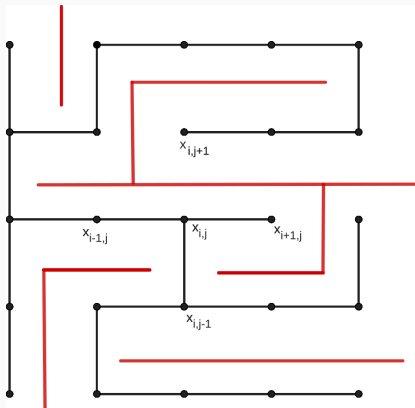





UAV inspection animations (from [Ivi+23]): portal , wind turbine , bridge 

[Ivi+23] S. Ivić et al. “Multi-UAV trajectory planning for 3D visual inspection of complex structures”. In: *Automation in Construction* 147 (2023), p. 104709

Regarding the poster that did not arrive to Japan



50x50 maze explored by 50 agents

Maze exploration (from [CIZ23]): 

References

- ▶ B.O. Koopman. “The theory of search. II. Target detection”. In: *Operations research* 4.5 (1956), pp. 503–531.
- ▶ S. Ivić, A. Andrejčuk, and S. Družeta. “Autonomous control for multi-agent non-uniform spraying”. In: *Applied Soft Computing* 80 (2019), pp. 742–760.
- ▶ S. Ivić. “Motion Control for Autonomous Heterogeneous Multiagent Area Search in Uncertain Conditions”. In: *IEEE Transactions on Cybernetics* (2020).
- ▶ S. Ivić, A. Sikirica, and B. Crnković. “Constrained multi-agent ergodic area surveying control based on finite element approximation of the potential field”. In: *arXiv preprint arXiv:2109.10756* (2021).
- ▶ B. Crnković, S. Ivić, and M. Zovko. “Fast algorithm for centralized multi-agent maze exploration”. In: *arXiv preprint arXiv:2310.02121* (2023).
- ▶ S. Ivić et al. “Multi-UAV trajectory planning for 3D visual inspection of complex structures”. In: *Automation in Construction* 147 (2023), p. 104709.
- ▶ L. Lanča, K. Jakac, and S. Ivić. “Model predictive altitude and velocity control in ergodic potential field directed multi-UAV search”. In: *arXiv preprint arXiv:2401.02899* (2024).